Evolutionary equilibria detection in non-cooperative games

Dan Dumitrescu, Rodica Ioana Lung, Tudor Dan Mihoc

Universitatea Babeş Bolyai Cluj Napoca

Abstract. An evolutionary approach for detecting equilibria in non-cooperative game is proposed. Apropriate generative relations (between strategies) are introduced in order to characterized game equilibria. The concept of game is generalized by allowing players to have different types of rationality. Experimental results indicates the potential of the proposed concepts and technique.

1 Introduction

Several types of equilibrium (solution) concepts have been proposed in non-cooperative Game Theory [7] [1] [3]. Each concept of equilibrium can be considered as expressing a different type of rationality. Within the present day approaches each equilibrium concept is addressed separately, meaning that in a particular game players interact accordingly to a unique equilibrium concept. Therefore only players guided by the same kind of equilibrium concept are allowed to interact. Our aim is to go beyond this paradigm in order to obtain a more realistic description.

A finite strategic game is defined by $\Gamma = ((N, S_i, u_i), i = 1, n)$ where:

- N represents the set of players, $N = \{1, ..., n\}$, n is the number of players;
- for each player $i \in N$, S_i represents the set of actions available to him, $S_i = \{s_{i_1}, s_{i_2}, ..., s_{i_m}\}$; $S = S_1 \times S_2 \times ... \times S_N$ is the set of all possible situations of the game;
- for each player $i \in N$, $u_i : S \to R$ represents the payoff function.

Let $U = \{u_1, ..., u_n\}$.

We denote by (s_{i_j}, s^*_{-i}) the strategy profile obtained from s^* by replacing the strategy of player i with s_{i_j} i.e.

$$(s_{i_j}, s^*_{-i}) = (s^*_1, s^*_2, ..., s^*_{i-1}, s_{i_j}, s^*_{i+1}, ..., s^*_1).$$

A strategy is called Nash equilibrium [5] if each player has no incentive to unilaterally deviate i.e. it can not improve the payoff by modifying its strategy while the others do not modify theirs. Informally we can state that these players behaviour is Nash-driven. We may also say that they play according to a Nash meta-strategy or they have a Nash type of rationality. In a similar manner we can speak about Pareto, stable state or other types of rationality.

A natural question is what happens if the players compete against each other relying on different types of rationality. A generalized game were agents are not uniform with respect to the rationality type is introduced. The type of rationality may be considered as reflecting the player interests, bias or subjectivity. For instance the players can be more or less cooperative, more or less competitive. In this way we can also allow players to be biased towards selfish or altruistic behaviour.

We assume the rationality type is described by an adequate meta-strategy concept. In a game players may assume different meta-strategies. The new paradigm offers a more realistic view and opens the possibility to further development in the Game Theory and significant applications. For instance multi-agent systems could benefit from the new approach.

The concept of generalized game with players characterized by several types of rationality is investigated. The new concepts are exemplified by considering a game were some players are Nash - and the other are Pareto-driven. An evolutionary technique for detecting the corresponding equilibrium for the generalized game is proposed.

2 Meta-strategy concept

Consider a game with n players. Each player i has a strategy s_i consisting of m_i actions (pure strategies). Consider that each player has a certain type of rationality. Let us denote by r_i the rationality type of the player $i = 1, .., n$.

A meta-strategy is a system

$$(s_1|r_1, s_2|r_2, ..., s_n|r_n),$$

where $(s_1, ..., s_n)$ is a strategy profile. A finite strategic generalized game is defined as a system by $G = ((N, M_i, u_i),\ i = 1, n)$ where:

- N represents the set of players, $N = 1, ..., n$, n is the number of players;
- for each player $i \in N$, M_i represents the set of available meta-strategies,
- $M = M_1 \times M_2 \times ... \times M_N$ is the set of all possible situations of the generalized game and $S = S_1 \times S_2 \times ... \times S_N$ is the set of all strategies;
- for each player $i \in N$, $u_i : S \to \mathbf{R}$ represents the payoff function.

Remark. In a generalized game the set of all possible meta-strategies may also be called the meta-strategy search space.

If $r_1 = r_2 = ... = r_n$, we say that the meta-strategy search space is *regular*. Otherwise it is said to be *irregular*.

In an irregular meta strategy space we may have different types of strategies expressing different types of rationality. It is natural to assume that the rationality type induces a particular type of equilibrium. We intend to explore the influence of irregularity on the detected equilibria. In this respect an evolutionary approach can be useful.

3 Generative relations in generalized games

In order to capture several kind of equilibria we relay on particular relations between meta-strategies. The intuition behind this assumption is that each type of equilibrium can be expressed by an appropriate *generative relation*. For example if we are interested in Pareto equilibrium, standard Pareto dominance relation is considered. Mixed equilibria can be also be detected. The problem is to find the appropriate generative relation for a certain type of equilibrium.

3.1 Pareto domination relation

In this section three generative relations are considered. Two of them correspond to Pareto and Nash equilibria. The third induces a new type of equilibrium which we called mixed Nash-Pareto equilibrium.

Let us consider two strategy profiles x and y from S.

Definition 1. The strategy profile x Pareto dominates the strategy profile y (and we write $x \ < P\, y$) if the payoff of each player using strategy x is greater or equal to the payoff associated to the strategy y and at least one payoff is strictly greater.

More formal we can write $x \ < P\, y$ iff

$$u_i(x) \geq u_i(y), \texttt{ for each } i = 1, .., n,$$

and there is some index j that

$$u_j(x) > u_j(y).$$

The set of all non-dominated strategies (Pareto frontier) represents the set of Pareto equilibria of the game.

3.2 Nash - ascendancy relation

Similar to Pareto equilibrium a particular relation between strategy profiles can be used in order to describe Nash rationality. This relation is called Nash-ascendancy (NA).

Definition 2. The strategy profile x Nash-ascends the strategy profile y, and we write $x \ < NA\, y$ if there are less players i that can increase their payoffs by switching their strategy from x_i to y_i then vice versa.

Let s^* be a strategy profile. We denote by $(s_{i_j}, s^*{}_{-i})$ the strategy profile obtained from s^* by replacing the strategy of the player i by s_{i_j} i.e.

$$(s_{i_j}, s^*_{-i}) = (s^*_1, s^*_2, ..., s^*_{i-1}, s_{i_j}, s^*_{i+1}, ..., s^*_n)$$

In [6] is introduced an operator

$$k : S \times S \rightarrow \mathbf{N},$$

$$k(y, x) = card\{i \in \{1, ..., n\} | u_i(x_i, y_{-i}) \geq u_i(y), x_i \neq y_i\}.$$

$k(y, x)$ denotes the number of players which benefit by switching from y to x.

Proposition 3. *The strategy x Nash-ascends y (x is NA-preferred to y), and we write $x \ < NA\, y$, if the inequality*

$$k(x, y) < k(y, x),$$

holds.

Definition 4. The strategy profile s^* is said to be non-Nash-overcome (non-NO) in S if there is no strategy $s \in S$, $s \neq s^*$, such that $s \ < NA\, s^*$.

According to [6] the set of all non-NO strategies from S equals the set of Nash equilibria.

This result proves that the Nash ascendancy is the generative relation for the Nash equilibrium.

3.3 NP - efficiency relation

Let us consider two meta-strategies

$$x = (x_1|r_1, x_2|r_2, ..., x_n|r_n),$$

and

$$y = (y_1|r_1, y_2|r_2, ..., y_n|r_n).$$

Let us denote by I_N the set of Nash biased players (N-players) and by I_P the set of Pareto biased players (P-players). Therefore we have

$$I_N = \{i \in \{1, ..., n\}|r_i = \text{Nash}\},$$

and

$$I_P = \{j \in \{1, ..., n\}|r_j = \text{Pareto}\}.$$

Let us introduce an operator E, measuring the relative efficiency of meta-strategies:

$$E : M \times M \to \mathbf{N},$$

defined as

$$E(x, y) = card(\{i \in I_N|u_i(x_i, y_{-i}) \geq u_i(y), x_i \neq y_i\} \cup \{j \in I_P|u_j(x) < u_j(y), x \neq y\}).$$

Remark. $E(x, y)$ measures the *relative efficiency* of the meta-strategy x with respect to the meta-strategy y.

The relative efficiency enables us to define a relation between meta-strategies.

Definition 5. Let $M_1, M_2 \in M$. The meta-strategy M_1 is more efficient than meta-strategy M_2, and we write $M_1 \ < E\, M_2$, iff

$$E(M_1, M_2) < E(M_2, M_1).$$

Efficiency relation can be easy generalized for n, $n > 2$, types of rationality.

In what follows we consider that efficiency relation induces a new type of equilibrium called *mixed Nash-Pareto equilibrium.*

The Nash-Pareto equilibrium defined by the efficiency relation is diffrent from the concept of Pareto-Nash equilibria introduced by [8]. The Pareto-Nash equilibria reffers to a pure Nash equilibrium is Pareto optimal i.e. the game admits no other pure Nash equilibrium for which each player has a strictly higher payoff.

4 Generalized two player game

Consider a two player non-cooperative game. Let r_i be the rationality type of player i. If $r_1 = r_2 =$ Nash then both players are Nash biased and the corresponding solution concept is the Nash equilibrium.

If $r_1 = r_2 =$ Pareto then both players are Pareto-biased and the corresponding equilibria are described by the set of non-dominated strategies (Pareto front).

We intend to explore the mixed case where the first player is Nash biased and the second one is Pareto biased, i.e.

$$r_1 = \text{Nash}, r_2 = \text{Pareto}.$$

Remark. In this case the sets I_N and I_P form a partition of the set $\{1, ...n\}$.

In order to detect the mixed Nash-Pareto equilibria of the generalized game an evolutionary approach may be used.

4.1 Detecting Mixed N-P equilibria in generalized games

Proposed technique for detecting equilibria in (generalized) games evolves a population of meta-strategies

Let us consider an initial population $P(0)$ of p meta-strategies for the generalized two player game. Each member of the population has the form

$$x = (s_1|r_1, s_2|r_2).$$

Pairs of meta-strategies are randomly chosen from the current population $P(t)$. For each pair a binary tournament is considered. The meta-strategies are compared by means of the efficiency relation. An arbitrary tie breaking is used if the two meta-strategies have the same efficiency. The winers of two binary tournaments are recombined using the simulated binary crossover (SBX) operator [10] resulting two offspring. Offspring population is mutated using real polinomial mutation [9], resulting an intermediate population P'. Population $P(t)$ and P' are merged.

The resulting set of meta-strategies is sorted with respect to the efficiency relation using a fast non-dominant sorting approach similar with [9]. For each meta-strategy M' the number expressing how many meta-strategies in the merged population are less efficient then M' is computed. On this basis the first p meta-strategies are selected from the merged population. Selected meta strategies represent the new population $P(t+1)$.

Let us remark that in the proposed technique selection for recombination and survival is driven by the efficiency relation. Therefore the population of meta-strategies is expected to converge towards the mixed Nash Pareto front. According to the proposed approach the members of this front represent the N-P equilibria of the generalized game.

The previous method is called *Relation based Equilibrium Detection (RED) Algorithm* and can be described as follows:

RED algorithm

S1. Set $t = 0$;
S2. Randomly initialize a population $P(0)$ of meta-strategies;
S3. Binary tournament selection and recombination for $P(t) \rightarrow Q$;
S4. Mutation on $Q \rightarrow P'$;
S5. Compute the rank of each population in member $P(t) \cup P'$ with respect to the efficiency relation. Order by rank $(P(t) \cup P')$;
S6. Rank based selection for survival $\rightarrow P(t+1)$;
S7. Repeat steps S3 - S6 until the maximum generation number is reached.

The evolutionary mechanism in this section is similar to that involved in NSGA2 algorithm [2] proposed by Deb et al. for the multi objective optimization.

5 Numerical experiments

In order to exemplify the proposed concepts the duopoly Cournot model is considered (see for instance [4]).

Let q_1 and q_2 denote the quantities of an homogeneous product - produced by two companies respectively. The market clearing price is

$$P(Q) = aQ,$$

where

$$Q = q_1 + q_2,$$

is the aggregate quantity on the market. Hence we have

$$P(Q) = \begin{cases} a - Q, & \text{for Q < a,} \\ 0, & \text{for Q} \geq \text{a.} \end{cases}$$

Let us assume that the total cost for the company i of producing quantity q_i is $C_i\ (q_i) = cq_i$. Therefore, there are no fixed costs and the marginal cost c is constant, $c < a$. Suppose that the companies choose their quantities simultaneously. The payoff for the company i is its profit, which can be expressed as:

$$\begin{aligned} \pi_i(q_i, q_j) &= q_i P(Q) - C_i(q_i) \\ &= q_i \left[a - (q_i + q_j) - c\right]. \end{aligned}$$

Several experiments have been performed for this game by using RED technique. The Cournot model with parameter $a = 24$ and $c = 9$ is considered in these numerical experiments.

Experiment 1. Nash - Nash rationality

Let us consider two players having the same rationality type $r_1 = r_2 = Nash$. In 10 runs the RED algorithm detects the Cournot's game unique Nash equilibrium point (5,5) and the corresponding payoff (25,25). The final population is depicted in Figure 1.

Experiment 2. Pareto - Pareto rationality

For $r_1 = r_2 = Pareto$ the RED population converges towards the Pareto front in 10 runs. The final population is depicted in Figure 1.

Fig. 1. Detected Nash point of equilibrium and Pareto front in 10 runs

Experiment 3. Nash - Pareto rationality.

Let us now consider the players having different kind of rationality, namely $r_1 = Nash$ and $r_2 = Pareto$. In 15 runs the population $P(t)$ of meta-strategies converges towards the front depicted in Figure 2. The set of payoff values are distributed from the payoffs of the Pareto front to the payoffs corresponding to the Nash equilibrium.

Fig. 2. Nash - Pareto front detected in 15 runs

Experiment 4. Pareto - Nash rationality

Let us now consider the case $r_1 = Pareto$ and $r_2 = Nash$. In 15 runs the population $P(t)$ of meta-strategies converges and the set of payoff values are distributed from the payoffs corresponding to the Nash equilibrium to the payoffs of the Pareto front.

In order to have a complete picture of detected equilibria, the results are represented together in Figure 4. An expected symmetry of the detected N-P and P-N fronts with respect to the first bisector is noted. The sets of detected strategies supply a discrete representation of the mixed Nash - Pareto equilibrium.

Fig. 3. Discrete representation of the pure and mixed equilibria in 15 runs

The average payoffs in the final population is given in Table 1 and Table 2. Some interesting remarks can be drawn from these tables:

(*i*) the Pareto players in the final population have better average payoffs in each game (respectivly the Nash players in the final population obtain poorer payoffs in each game);

(*ii*) the Pareto players take an advantage in the mixed meta-strategies (N-P, P-N) compared with (P-P) meta-strategy ;

(*iii*) the Nash player are clearly handicapped in the mixed equilibria (meta-strategies) compared with the Nash-Nash pure equilibrium (meta-strategy);

(*iv*) The average payoff in mixed equilibria (N-P, P-N) is between those of the pure equilibria (N-N, P-P);

(*v*) experiments cannot differentiate between the payoffs associated to N-P and P-N mixed equilibria.

Table 1. Average payoff and standard deviation in the final population of 100 meta-strategies after 30 generations.

Meta-strategy type							
Nash - Pareto		Pareto - Nash		Nash - Nash		Pareto - Pareto	
Player rationality	Avg. payoff (St.Dev.)	Player rationality	Avg. payoff (St.Dev.)	Player rationality	Avg. payoff (St.Dev.)	Player rationality	Avg. payoff (St.Dev.)
Nash	13.5 (7.36)	Nash	12.5 (7.28)	Nash	25.00 (0.08)	Pareto	27.60 (16.8)
Pareto	41.00 (8.81)	Pareto	42.2 (8.71)	Nash	25.00 (0.08)	Pareto	28.50 (16.8)
Total Payoff							
54.50		54.70		50.00		56.10	

6 Conclusions and future work

A concept of generalized game, involving several types of rationality is proposed. The type of rationality is captured by the concept of meta-strategy. The corresponding solution concept (equilibrium) is induced by a generative relation between meta-strategies. This allows the combination of different types of equilibria in a game.

An evolutionary technique for detecting an approximation of the generalized equilibria is developed. The idea are exemplified for a Cournot game with two types of rationality.

Table 2. Average payoff and standard deviation in the final population of 200 meta-strategies after 100 generations.

Meta-strategy type							
Nash - Pareto		Pareto - Nash		Nash - Nash		Pareto - Pareto	
Player rationality	Avg. payoff (St.Dev.)	Player rationality	Avg. payoff (St.Dev.)	Player rationality	Avg. payoff (St.Dev.)	Player rationality	Avg. payoff (St.Dev.)
Nash	12.86 (7.53)	Nash	12.98 (7.40)	Nash	25.00 (0.00)	Pareto	28.27 (16.4)
Pareto	41.76 (9.19)	Pareto	41.49 (8.94)	Nash	25.00 (0.00)	Pareto	27.91 (16.4)
Total Payoff							
54.47		54.62		50.00		56.19	

Experimental results offer an inside view of the problems arising when two different type of equilibria are considered in the same game. Results also indicate the potential of the proposed technique. Future work will address generalized games having more than two rationality types and more players.

References

1. Bade, S., Haeringer, G., Renou, L.: *More strategies, more Nash equilibria*, Working Paper 2004-15, School of Economics University of Adelaide University, 2004.
2. Deb, K., Agrawal, S., Pratab, A., Meyarivan, T.: *A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II*, Marc Schoenauer, Kalyanmoy Deb, Günter Rudolph, Xin Yao, Evelyne Lutton, Juan Julian Merelo, and Hans-Paul Schwefel, editors, Proceedings of the Parallel Problem Solving from Nature VI Conference, Paris, France, 2000. Springer, Lecture Notes in Computer Science, 1917, 849-858
3. D. McKelvey, R., D., McLennan, A.: *Computation of equilibria in finite games*, In H. M. Amman, D. A. Kendrick, and J. Rust, editors, Handbook of Computational Economics, Elsevier,1996.
4. Lung, R. I., Muresan, A. S., and Filip, D. A.: *Solving multi-objective optimization problems by means of natural computing with application in finance*, In Aplimat 2006 (Bratislava, February 2006), pp. 445-452.
5. Nash.,J.,F.: *Non-cooperative games*, Annals of Mathematics, 54:286-295, 1951.
6. Lung,R.,I., Dumitrescu, D.: *Computing Nash Equilibria by Means of Evolutionary Computation*, Int. J. of Computers, Communications & Control, 2008, 364-368
7. Osborne,M. J., Rubinstein, A.: *A Course in Game Theory*, MIT Press, Cambridge, MA, 1994

8. Maskin, E. : *The theory of implementation in Nash equilibrium:A survey*, in: L. Hurwicz, D. Schmeidler and H. Sonnenschein, eds., Social Goals and Social Organization (Cambridge University Press), 1985,173-204
9. Deb, K., Agrawal, S., Pratab, A., Meyarivan, T.: *A Fast and Elitist Multi-Objective Genetic Algorithm: NSGA-II* KanGAL Report No. 200001, Indian Institute of Tehnology Kanpur, 2000
10. Deb, K., Beyer, H.: *Self-adaptive genetic algorithms with simulated binary crossover*, Complex Systems,1995, vol.9, 431–454

This article was processed using the LaTeX macro package with LLNCS style